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AN APPLICATIONS OF CONTROLLED JUMP MODEL IN FINANCE

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ABSTRACT

The purpose of this paper is to identify the problem formulation of controlled model with jump process.

KEYWORDS: random differential equation, jump process, control.

INTRODUCTION

Both types of random differential equations (continuous and with jump component) have great importance in studying models from natural sciences and from economics as they used to describe the movement of the stock price, in mathematical finance the applications of the random differential equations including jump appear in the cause when the stock price make a sudden shift, to model this, one would like to represent the stock price by a process that has jump. The random control problems that described by system with jump component play an important role in investment. The general problem of this kind of processes with jump is considered in the literature, for example see [1], [2].

Our starting point is a random_experiment modeled by a complete filtered probability space $\left(\Omega, F, \{F_t\}_{t\in T}, P\right)$ where Ω the set is all possible outcomes of this experiment, F is the σ-algebra of events, $\{F_t\}_{t\in T}$ is the filtration which represent the record of information about the experiment over time $[0,T]$, and P is the true or physical probability measure.

Random Process: Random process indexed by T is collection of random variable defined by the map $X : \Omega \times T \to R^n$ such that $\forall\ t \in T\ ,\ \ \omega \to X(\omega, t)$ is measurable

On the measurable_space $(\Omega\ ,\ \mu)$, the process X_t is said to be adapted to the filtration $\{F_t\}_{t\in T}$ if the random variable $X_t : \Omega \to R^n$ is a $(F_i\ ,\ \mu)$-measurable_function for each $t \in T$.

Càdlàg function: A function $f : E \to M$ is called a càdlàg function if for every $t \in E$.

- the left limit $f(t-) := \lim_{s \uparrow t} f(s)$ exists; and
- the right limit $f(t+) := \lim_{s \downarrow t} f(s)$ exists and equals $f(t)$.

That is, f is right-continuous with left limits.

If X_t is a process that is right continuous with left limit , we set $X_{t-} = \lim_{s\to t, s<t} X_s$ and $\Delta X_t = X_t - X_{t-}$,

thus ΔX_t is the size of jump of the process X_t at time t , and zero when there is no jump.

Sample path: for fixed $\omega \in \Omega$, the sample path of the random process is the map $t \to X(t,\ \omega)\ ,\ t \in T$.

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A process X_t is said to be continuous time random process if it is sample path is continuous function otherwise X_t is called discontinuous random process (or process with jump component).

A jump process defines as a type of random process that has discrete movements, called jumps, rather than small continuous movements, where the notion of jump is common in mathematics. The jump process usually denoted by N_t .The simplest jump process we have is the standard Passion Process.

Levy process: A Cadlag, adapted, real valued random process X_t , $t \geq 0$ is said to be a Lévy process if it satisfies the following:

1. $X_0 = 0$ almost surely
2. Independence of increments: For any $0 < t_1 < t_2 < t_3 < \cdots t_n < \infty$ $X_{t_2} - X_{t_1}, X_{t_3} - X_{t_2}, \cdots, X_{t_n} - X_{t_{n-1}}$ are independent
3. Stationary increments: For any $s < t$, $X_t - X_s$ is equal in distribution to X_{t-s}
4. Continuity in probability: For any $\varepsilon > 0$ and $t \geq 0$ it holds that $\lim_{h \to 0} P\left(\left|X_{t+h} - X_t\right| > \varepsilon\right) = 0$

Theorem: Let X_t be levy process, then X_t has unique cadlag version which is also Levy process, see [3]

The sum of a linear drift Brownian motion and a compound Poisson process is levy process, often called a jump-diffusion process. We shall call it Levy jump diffusion process, since there exist jump-diffusion processes which are not levy processes.

Stopping times: A random variable τ with values in $[0,1]$is an $\{F_t\}-$ stopping time if $\{\tau < t\} \in F_t$, $\forall t \geq 0$. Stopping time usually defined as mechanism for deciding whether to continue or to stop a process.

Let T be the class of $F\_$ stopping times satisfying $\tau_s \in T$, for all $\tau \in T$, then $\tau \leq \tau_s$ a.s

Random differential equation is differential equation in which one or more of the terms is random process (continuous or with jump component), resulting in a solution which is itself random process, see [4].

1-Dimensional Ito formula: Let X_t be 1- dimensional Ito processes given by: $d X_t = u(t)\, dt + v(t)\, d B_t$. Let $g(x,t) \in C^2\left([0,\infty) \times R\right)$, then $Y_t = g\left(t, X_t\right)$is again Ito processes and

$$d\ Y_t = \frac{\partial g\left(t, X_t\right)}{\partial t} dt + \frac{\partial g\left(t, X_t\right)}{\partial x} d X_t + \frac{1}{2} \frac{\partial^2 g\left(t, X_t\right)}{\partial x^2} (d X_t)^2.$$

For the proof see [4].

Itô’s formula is the fundamental theorem of random calculus.

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The measurable deterministic function $u : R^n \to U$ that control the general solution of any given differential equation so as to give the maximum or minimum value is called control. $U = \{u \; , \; u : R^n \to U\}$ is the set of all controls usually known as control region .

Let $\{\tau_j\}, \; j = 1,2,3,\cdots$ be the infinite joint sequence of stopping times such $\tau_j < \infty \; a.s$ and $\tau_j < \tau_{j+1}$ and let $\varsigma_1 , \varsigma_2 , \varsigma_3 , \ldots$ be the non- negative sequence of impulse values , such that for each $j = 1,2,3,\cdots$, ς_j takes values in $Z \subset R$, and ς_j is measurable with respect to the filtration F_{τ_j} then we define the impulse control v as the double sequence $v = \left((\tau_j , \varsigma_j) , \; j = 1,2,3,\cdots , M \right)$ where M is random variable taking values in $[\, 0,1,2,3,\cdots) \cup \{\infty\}$.

W says that the impulse control v is admissible if the following two conditions are hold:

1- The corresponding state process $X = X^v$ exists and is unique.

2- With probability one, either $M(\omega) < \infty$ or if $M(\omega) = \infty$ then

$$\lim_{j \to \infty} \tau_j(\omega) = \infty$$

Let $V = V_T$ and Let $v^k = \left((\tau_1 , \varsigma_1) , (\tau_2 , \varsigma_2) , \ldots\ldots , (\tau_k , \varsigma_k) \right)$ to be the first k times, and impulses. Denote by v^0 to empty impulse control with no intervention on $[0,\infty]$ with $\tau_0 = 0$.

Define the space $W = W_T$ of admissible combined random controls to be $v = \left((\tau_1 , \varsigma_1) , (\tau_2 , \varsigma_2) , \ldots\ldots , \right) \in U \times V_T$ satisfying

$$\left(\tau_k(\omega) , \varsigma_k\right) = \left(\tau_s , \varsigma\right) \; , \; if \;\; \tau_k(\omega) = \tau_s$$

The pair $w = (u,v)$ where $u = \in U$ is called the combined random control.

See [5],[6].

Assume that the problem is described by system with jump component of the form:

$$d X(t) = \mu X(t) \, dt + \sigma X(t) \, d B_t + \int_0^\infty h(X_t , y) \, \hat{N}(d t , d y) \; , \; X(0) = x > 0 \qquad (1)$$

Where $\mu(X), \sigma(X)$ and $h(X)$ are real valued functions and B_t is

1-dimensional Brownian motion with respect to $\{F_t\}$ and $\hat{N}(dt, dy)$ is the

Compensated Poisson random measure given by:

$\hat{N}(d t , d y) = N(d t , d y) - d t \, \pi(d y)$ where $\pi(d y)$ is the Levy measure associated to N .

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For integrablity reasons assume that:

$$\int_{-1}^{\infty} \left(1 \wedge y^2\right) \pi(d\, y) < \infty \qquad , \qquad \int_{-1}^{\infty} y\, \pi(d\, y) < \infty$$

We also assume that the size of a jump is greater than -1, so that $X(t)$

remain non- negative for all $t \geq 0$ a.s

PROBLEM FORMULATION

Consider the dynamic in (1) , and the assumptions mentioned above ,suppose that at any time t and any state we can intervene by giving the system $X(t)$ an impulse $\varsigma \in Z \subset R^l$, the result of the impulse is that the state jumps immediately suppose from $X(t^-) = y$ to a new state $X(t) = \Gamma(y, \varsigma)$ where $\Gamma : R^k \times Z \to R^k$ is given function.

That is when $v = ((\tau_1, \varsigma_1), (\tau_2, \varsigma_2), \ldots\ldots,)$ is applied to the system $X(t)$, then the dynamics of the controlled jump model $X^{(v)}(t)$ can be described by:

$$\begin{cases} d\, X^{(v)}(t) = \mu\left(X^{(v)}(t)\right) d\, t + \sigma\left(X^{(v)}(t)\right) d\, B_t \\ \qquad + \int h\, (X^{(v)}(t), y)\, \hat{N}(dt, d\, y), \quad \tau_j < t < \tau_{j+1} \leq T^* \\ X^{(v)}\left(\tau_{j+1}\right) = \Gamma\left(X^{(v)}\left(\tau^-{}_{j+1}\right), \varsigma_{j+1}\right), \quad j = 0, 1, 2, \cdots, \quad \tau_{j+1} < T^* \end{cases}$$

Where T^* is defined by: $T^*(\omega) = \lim_{R \to \infty} \left(\inf \left\{t > 0\, ;\, \left| X^{(v)}(t) \right| \geq R \right\}\right) \leq \infty$.

Let $Q^x = Q^{x,v}$ denote the law of the random process $X^{(v)}$ starting at $X^{(v)}(0) = x$.

Let $D \subset R^k$ be a given closed set which will be interpreted as the domain in the sense that we are only interested in the process up to the first exit time from D such that $D \subset \left(\overline{D}^0\right)$, where D^0 denotes the interior of D and $\left(\overline{D}^0\right)$ its closure.

Suppose that the profit rate when the system is in state x is $f(x)$ where $f : S \to R$ is given function. Denote to the boundary of D by ∂D.

Let $g : \partial S \to R$ be a given measurable bequest function.

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Suppose also that the utility of performing an intervention with impulse $\varsigma \in Z$ when the system is in state x is $K(y, \varsigma)$ where $K : S \times Z \to R$ is given function.

Suppose that $\forall \; y \in R^k$:

$$E^y\left[\int_s^T \left| f\left(Y(t)\right)\right| dt\right] < \infty \;, E^y\left[\int_s^T \left| g\left(Y(T)\right)\right| \chi_{\{T<\infty\}}\right] < \infty \,, E^y\left[\sum_{\tau_j < T} \left| K\left(Y^{(v)}\left(\tau^-{}_j\right), \varsigma_j\right)\right|\right] < \infty$$

When we applying $v = ((\tau_1, \varsigma_1), (\tau_2, \varsigma_2), \ldots\ldots,) \in V$ to the system then the total expected profit utility is given by:

$$J^v(y) = E^x\left[\int_s^{\hat{T}} f\left(X^{(v)}(t), u_i\right) dt + \sum_{\tau_j < T} K\left(X^{(v)}\left(\tau^-{}_j\right), \varsigma_j\right)\right] < \infty$$

Where E^x denote the expectation with respect to Q^x.

Define $\Phi(x)$ to be the value function (the function that optimizing the cost over all admissible controls) , and $v^* \in V$ to be the optimal impulse control .

Then the problem formulation of the controlled jump model is of the form:

$$\Phi(x) = \sup\left\{ J^v(x) \,, \quad x \in R^n \,, \quad v \in V \right\} = J^{(v^*)}(x) \,.$$

REFERENCES

[1] Thomas G. Kurtz: Lectures on Stochastic Analysis. University of Wisconsin- Madison , WI 53706-1388 September 2001
[2] Bernt OKsendal: Stochastic Differential Equations. Springer - Verlag1998
[3] P. Potter: Stochastic Integration and Differential Equation. Springer- Verlag 1999.
[4] Øksendal, B. K. Stochastic differential equations: An introduction with applications. Berlin: Springer(1998).
[5] Nils Cher . Framstate , Bernt Oksndal and Agent Sulem: Optimal Consumption and Portoflio in Jump Diffusion Market with Proportional Transaction Castes, University of Oslo.2000
[6] J. Klamka, Controllability of Dynamical Systems, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1991.